multiplied by a -ive number ...

Complex Numbers

The numbers of the form $x + iy$ where $x, y \in R$ are called complex numbers.

Here.

x = Real part of complex number

y = Imaginary part of complex number

Note: Every real number is a complex number with zero as it imaginary part.

Operation on Complex Numbers

If a, b, c, d, k are real numbers then following hold for complex numbers.

1. $a + bi = c + di \Rightarrow a = c \wedge b = d$
2. $(a + bi) + (c + di) = (a + c) + (b + d)i$
3. $k(a + bi) = ka + kbi$
4. $(a + bi) - (c + di) = a + bi + (-c - di) = (a - c) + (b - d)i$
5. $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Complex Numbers as Ordered Pairs

Let C be the set of all ordered pairs belonging to $R \times R$ which satisfy the following properties.

1. $(a, b) = (c, d) \Rightarrow a = c \wedge b = d$
2. $(a, b) + (c, d) = (a + c, b + d)$
3. $(a, b) - (c, d) = (a, b) + (-c, -d) = (a - c, b - d)$
4. $k(a, b) = (ka, kb)$
5. $(a, b)(c, d) = (ac - bd, ad + bc)$

then C is called the set of complex numbers.

Conjugate Complex Numbers

Complex numbers which have the same real parts and whose imaginary parts differ in sign only a called conjugate of each other

e.g $a + bi$ and $a - bi$ are conjugate to each other.

Note:

1. A real number is self conjugate.
2. A number which has zero as its imaginary part is self conjugate.

Properties of the Fundamental Operations on Complex Numbers.

The set 'C' of complex numbers form a field with

1. $(0, 0)$ as the additive identity of C.
2. Every complex number (a, b) has the additive inverse $(-a, -b)$
   i.e $(a, b) + (-a, -b) = (0, 0)$
3. $(1, 0)$ is the multiplicative identity of C. i.e.
   $(a, b) . (1, 0) = (a . 1 - b . 0, b . 1 + a . 0) = (a, b) = (1, 0)(a, b)$
4. Every non-zero complex number {i.e. $(a, b) \neq (0, 0)$} has multiplicative inverse.

   Multiplicative of $(a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right)$

   i.e. $(a, b)\left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right) = (1, 0)$
5. $(a, b)[(c, d) \pm (e, f)] = (a, b)(c, d) \pm (a, b)(e, f)$

Note: The set C does not satisfy the order axioms. In fact there is no sense in saying that one complex num is greater or less than another.

A Special Subset of C

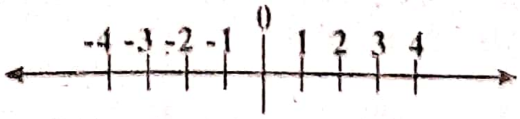
Consider the subset of C whose elements are of the form $(a, 0)$.

Let $(a, 0), (c, 0) \in C$

1. $(a, 0) + (c, 0) = (a + c, 0)$
2. $k(a, 0) = (ka, 0)$
3. $(a, 0) \times (c, 0) = (ac, 0)$
4. Multiplicative inverse of $(a, 0)$ is $\left(\frac{1}{a}, 0\right), a \neq 0$

The Real Line

The real line is



and any number which lies on it is called real number.

Cartesian Product

Let A and B be two non empty sets then Cartesian product of A and B is denoted by $A \times B$ and defined as $A \times B = \{(x, y) / x \in A \wedge y \in B\}$.

Ordered Pairs

The members of a Cartesian product are called ordered pairs.

Cartesian Plane: **(Federal 2015, Rwp 2016)**

The Cartesian product $R \times R$ is called the Cartesian plane. Where R is the set of real numbers.

Real Plane:

The geometrical plane on which coordinate system has been specified is called the real plane or coordinate plane.

Note: There is (1 – 1) correspondence between the elements of $R \times R$ and points of the plane.

Coordinates of a Points **(Sgd 2015, Lhr 2015 G II)**

If a point A of the coordinate plane corresponds to the ordered pair (a, b) then a, b are called the coordinates of A. a is called x– coordinate or abscissa and b is called y coordinate or ordinate.

Note: Corresponding to every $(a, b) \in R \times R$ there is one and only one point in the plane and vice versa.

Geometrical Representation of Complex Numbers

We represent complex numbers by points of the coordinate plane. Every complex number is represented by one and only one point of coordinate plane and every point of plane represent one and only on complex number. In this case components of complex number are the coordinates of the point representing it we call x – axis as real axis and y–axis as imaginary axis and coordinate plane is called complex plane or plane.

Argand Diagram:

The figure representing one or more complex numbers on the complex plane is called an argand's diagram.

Points on x-axis represent real numbers and points on the y-axis represent imaginary numbers.

Modulus of Complex Number

The modulus of a complex number $x + iy$ is denoted by $|x + iy|$ and is defined as

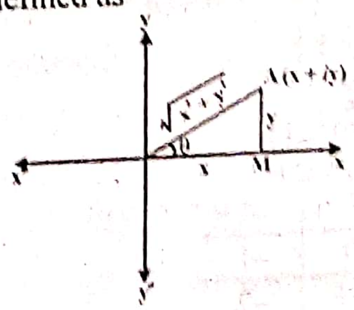
$$|x + iy| = \sqrt{x^2 + y^2}$$

If $\quad Z = x + iy$

then $\quad |Z| = \sqrt{x^2 + y^2}$

OR

The modulus of a complex number



is the distance from the origin of the point representing the number.

Palor Form of Complex Number

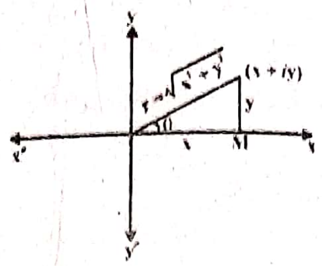
Let $z = x + iy$

From diagram $x = r\cos\theta$ $y = r\sin\theta$ where $r = |z|$

θ = argument of z $\therefore$ $x + iy = r\cos\theta + r\sin\theta$ where

$r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$. This is called

Polar form of complex number.



De Moivre's Theorem

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta) \qquad \forall\, n \in Z.$$

EXERCISE 1.1

Express each of the following complex numbers in the polar form (**Problems 1 – 6**):

1. $-\sqrt{3}+i$
2. $-i$
3. $-1-\sqrt{3}\,i$
4. $-1+i$
5. $(-2+2i)(1-i)$
6. $\dfrac{-34\,i}{5-3\,i}$

Express the given complex number in cartesian form and plot on an Argand diagram (**Problems 7 – 10**):

7. $2\text{ cis}\left(\frac{\pi}{6}\right)$
8. $5\text{ cis}\left(\frac{3\pi}{4}\right)$
9. $\sqrt{3}\text{ cis}\left(\frac{7\pi}{6}\right)$
10. $\dfrac{5\text{ cis }(\pi/3)}{2\text{ cis }(\pi/2)}$

11. Find $|z|$ where

(i) $z=-2i(1+i)(2+4i)(3+i)$ (ii) $z=\dfrac{(3+4i)(-1+2i)}{(-1-i)(3-i)}$